Numerical investigation of feedback control of thermocapillary instability

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Control of oscillatory thermocapillary convection in an annular geometry with a horizontal free
surface is investigated by means of a numerical simulation. The objective is to suppress oscillations
using a feedback opposition control. The temperature is measured at certain positions on the
interface, this signal is amplified and used to apply local heating on the free surface. Many features
of the controlled system observed in previous experiments could be reproduced by the simulation.

The numerical simulation allows us to clarify the picture of the spatial structures of the controlled
oscillation, which was not accessible in the experiments. In addition to what was found in the
previous experiments, the present simulations also permit us to investigate the importance of the
positioning of sensors and heaters, and the influence of the properties of the heaters. © 2005
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I. INTRODUCTION

In the refining process of crystal growth, the fluid physics of the melt is of great importance since the flow may have a strong impact on the quality of the final product. The fluid motion is mainly caused by thermocapillary and buoyancy forces, and sometimes electromagnetic forces. Especially, for the floating-zone technique, as it is suggested for space processing under microgravity, the main interest has been the thermocapillary convection.

One of the detrimental problems in the floating-zone technique are so called striations, inhomogeneous distribution of chemical compounds, and dopants appearing in the final single crystal. Since this was found to be due to the time-dependent oscillatory state of the thermocapillary convection, a number of extensive works have been reported.

A large part of the work reported in the literature has focused on simplified models where generic flows seen in the floating-zone are realized. The most common model to date is an axisymmetric geometry called the half zone, hereafter referred to as HZ. In HZ, a liquid drop is held between two coaxial rods maintained at different temperatures to impose an axial temperature gradient on the free surface. The first experimental detections of the oscillation were realized by Schwabe and Scharmann and Chum and Wuest in this type of geometry which motivated works in linear stability analyses, numerical simulations and experiments to study the onset instability characteristics and wave structures. Later, supercritical behaviors of the oscillation were revealed by both experiments and numerical simulations.

One of the difficulties in experimental investigations of HZ is the curvature of the free surface whose shape is known to have a strong influence on the flow. For the sake of better quantitative analyses and thus comparison with numerical simulations, another axisymmetric geometry, annular configuration (hereafter referred to as AC) was first suggested by Kamotani et al. The system is an open cylindrical container filled with a liquid to have a flat-free upper surface. A heated pipe with a prescribed temperature is located on the axis of the container. Thermocapillary convection is thus driven by imposing a radial temperature gradient on the flat-free surface. In this geometry, a flat-free surface can be easily realized and maintained. During the last decade, a series of extensive studies was carried out by the group of Kamotani and Ostrach including microgravity experiments in space. Lavalle and Ostrach reported detailed comparisons between experiments and simulations, as well as velocity fields obtained experimentally using particle image velocimetry.

Detailed experiments in microgravity and simulations for the AC have further elucidated the selection of the spatial structure and the azimuthal wavenumber of the oscillatory modes. The Prandtl number was 6.8, but the aspect ratio of the annular domain was varied by one order of magnitude, to include very shallow layers. It is found that the azimuthal wavenumber and the spatial complexity of the modes increases as the depth of the layer is decreased. A somewhat different geometry was studied by Sim and Zebib, which studied a dish filled with liquid, i.e., an AC without the central heated pipe. The thermal gradient is instead driven by an applied heat flux to the free surface, and cooling at the rim. Results are given for a Prandtl number of 30, and free surfaces with different curvatures. The results qualitatively resemble those found for AC with the appearance of traveling or standing waves as the thermocapillary Reynolds number exceeds a critical value.

Even if axisymmetric base flows in HZ and AC are qualitatively similar, there are some differences. When the surface flow is convective inertial, the vortex core is pulled towards the hot corner in HZ whereas the core stays in the center or is pushed towards colder corner in AC. Kamotani et al., from their observations, suggested that, in HZ and AC, the overall flow is mainly driven in the hot corner region and the viscous bulk region, respectively. Based on this idea,
they performed scaling analyses of characteristic variables in HZ and AC differently and saw good agreement with the results from two-dimensional (2D) numerical simulations. It should be noted that the analysis was carried out in the range of Marangoni numbers (Ma) equivalent to the onset value for oscillatory flows.

The ultimate industrial interest here is to produce stratification-free crystals, and one way would be to use feedback control to suppress the flow oscillations that are thought responsible. With this industrial motivation there have been a few studies recently of control of the convective thermocapillary instability.

The possibility to stabilize the thermocapillary instability by locally altering the heat flux on the surface was first demonstrated experimentally by Petrov et al.\textsuperscript{20,21} A nonlinear control was performed to stabilize the oscillation in a half-zone model by using local temperature measurements close to the free surface and modifying the temperature at different local locations. They have constructed a look-up table based on the system’s response to a sequence of random perturbations. A linear control law using appropriate data sets from the look-up table was computed. The control law was updated at every time step to adapt the control law to the nonlinear system. Using one sensor/actuator pair, a successful control was observed at the sensor location for Ma \approx 17 750; however, infrared visualization revealed the presence of standing waves with nodes at the feedback element and the sensor. This was resolved by adding a second sensor/actuator pair which allows the control to damp out both waves propagating clockwise and counterclockwise, thus standing waves. The performance of the control was reported for a fixed Ma \approx 15 000, where the critical value was Ma_c \approx 14 000.

This was followed by Benz et al.\textsuperscript{22} who applied control to a hydrothermal wave in a shallow fluid layer. The temperature signal and the phase information sensed by thermocouples near the cold end of the layer was fed forward to control a laser which heated the downstream fluid surface along a line.

For an annular configuration, Shiomi et al.\textsuperscript{23,24} applied active feedback control based on a simple cancellation scheme. Active control was realized by locally modifying the surface temperature using the local temperature measured at different locations fed back through a simple control law. Using two sensor/actuator pairs, a significant attenuation of oscillation was observed in a range of Ma, with the best performance in the weakly nonlinear regime. Applying the control to an oscillation with azimuthal wavenumber of 3 (mode 3), in the regime with weak nonlinearity, the oscillation was suppressed to the background noise level. The experiments also revealed the limit of the control. When Ma is about 15% above the critical value, control fails to achieve the complete suppression of the oscillation, though a significant attenuation is still achieved. The loss of control is accompanied with an increase in the amplitude of the first overtones and a modulation in the controlled signal which may suggest the appearance of another mode triggered by the control.

Recently, with the similar idea of the cancellation scheme, but in a half-zone model, linear and weakly nonlinear control of the oscillatory thermocapillary convection was reported by Shiomi et al.\textsuperscript{25} The experiment utilizes an unit aspect ratio liquid bridge where the most dangerous mode has an azimuthal wavenumber of 2 with absence of control. The performance of control was quantified by analyzing local temperature signals and the flow structure was simultaneously identified by flow visualization. With optimal placements of sensors and heaters, proportional control can raise Ma_c by more than 40%. The amplitude of the oscillation can be suppressed to less than 30% of the initial value up to 90% of Ma_c. The proportional control was tested for a period doubling state to stabilize the oscillation to a periodic state. Weakly nonlinear control was applied by adding a cubic term in the control law to improve the performance of the control and to alter the bifurcation characteristics of the system.

In the current study, we will study a few representative cases of control using numerical simulations. In addition to what was found in the previous experiments, the present simulations also permit us to investigate the importance of the positioning of sensors and heaters, and the influence of the properties of the heaters. We will also study the spatiotemporal decomposition of the modes that appear, as Fourier decompositions of the modes are easily accessible in a numerical simulation, as opposed to the experiments. It will be of particular interest to investigate how modes with different spatial structures are suppressed or amplified, depending on the heater positioning, heater size, etc.

II. GOVERNING EQUATIONS

As shown in Fig. 1, the geometry of the system is an open cylindrical container with radius \( R \) and depth \( H \) filled with liquid, with a planar-free upper surface. A central coaxial cylinder of radius \( R_h \) provides an inner radial boundary to the annular fluid volume. Thermocapillary convection is driven by a radial temperature gradient on the horizontal free surface, which is maintained by raising the temperature of the inner cylinder above that of the outer wall.

The aspect ratio \( A_r \equiv H/R \), was kept at unity. The ratio of the inner to outer radius of the annulus, \( H_r \equiv R_r/R \), is \( H_r = 0.21 \).

The bottom thermal boundary condition is adiabatic. The
surface tension is considered to be a linearly decreasing function of the temperature,
\[ \Gamma = \Gamma_0 - \gamma(T - T_0), \]
where the surface tension coefficient \( \gamma \) has a positive constant value. This means that the flow will be driven outwards on the free surface.

The fluid is treated as a 3D incompressible Newtonian liquid. Therefore, the flow is governed by the incompressible Navier–Stokes equations, energy equation, and continuity equation. In a zero-gravity environment, the governing equations are
\[ \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\nabla p + \frac{\Pr}{\Ma} \nabla \cdot (\nabla \mathbf{u} + \nabla \mathbf{u}^T), \]
\[ \frac{\partial \theta}{\partial t} + (\mathbf{u} \cdot \nabla)\theta = \frac{1}{\Ma} \nabla \cdot (\nabla \theta), \]
\[ \nabla \cdot \mathbf{u} = 0. \]
These equations have been nondimensionalized using the length \( R \), temperature difference \( \Delta T \), velocity scale from the thermocapillary stress balance,
\[ U = \frac{\gamma \Delta T}{\mu}, \]
and time scale
\[ t = \frac{R^3}{U}, \]
where \( \mu \) is the dynamic viscosity. The nondimensional parameters appearing above are the Marangoni number \( \Ma \) and the Prandtl number \( \Pr \) defined as
\[ \Ma = \frac{\gamma \Delta TR}{\mu \alpha}, \]
\[ \Pr = \frac{\nu}{\alpha}, \]
where \( \alpha \) and \( \nu \) are the thermal diffusivity and kinematic viscosity, respectively.

The boundary conditions for velocity are no slip on solid walls and a thermocapillary stress on the horizontal free surface. The thermal boundary conditions are that the temperatures of the inner and outer cylinders are prescribed at different values, thus enforcing a radial temperature gradient. The bottom is assumed thermally insulated. At the free surface an insolar unspecified heat flux is present. This heat flux \( q \) will below be set to account for the action of the heating elements that provide the control:
\[ \mathbf{u} = 0, \quad \theta = 1 \quad \text{at} \quad r = H_r, \]
\[ \mathbf{u} = 0, \quad \theta = 0 \quad \text{at} \quad r = 1, \]
\[ \frac{\partial v}{\partial z} = \frac{1}{r} \frac{\partial \theta}{\partial \phi}, \quad v = 0, \quad \frac{\partial \theta}{\partial z} = \frac{\partial \theta}{\partial r} = q(r, \phi, t) \quad \text{at} \quad z = A_r, \]
Here the equations have been formulated in terms of \( \Ma \). However, in the following we will frequently be referring to the overcritical parameter \( \epsilon \), defined as
\[ \epsilon = \frac{\Ma - \Ma_{cr}}{\Ma_{cr}}. \]
Here \( \Ma_{cr} \) denotes the critical Marangoni number for onset of oscillations. Once \( \Ma_{cr} \) is specified, \( \epsilon \) thus carries the same information as \( \Ma \), but it is a more descriptive parameter here where we are interested in properties of supercritical oscillations.

III. NUMERICAL METHOD

A finite element method in the \((r,z)\)-plane combined with a pseudospectral method in the azimuthal direction was used to solve the equations in cylindrical coordinates. A Galerkin approach is adopted to formulate the discrete equations. The solutions are expanded in azimuthal Fourier modes. For each mode, the equation system is solved in the 2D \((r,z)\) plane using triangular elements with quadratic base functions for the velocity and temperature, and piecewise linear functions for the pressure (P2P1). On computing the nonlinear terms for the \( N_\phi \) azimuthal planes, dealiasing was done by computing the nonlinear terms for \( 2 \times N_\phi \) planes and filtering the first \( N_\phi \) modes.

The time discretization is done using a semi-implicit scheme for the viscous terms and the nonlinear advection terms, respectively. The pressure is decoupled from the velocity computations by using a projection method. With this implementation, the resulting linear equation system is solved by a conjugate gradient method. The explicit treatment of the convection of the nonlinear terms impose a restriction on the timestep; a Courant–Friedricks–Lewey condition needs to be satisfied for numerical stability. The time necessary to simulate 1 s in physical time for \( \Ma \) well over the critical Marangoni number \( \Ma_{cr} \) is typically about 4 h on a PC with AMD Athlon MP2000.

The finite element method computation of the \((r,z)\) plane was coded using femLego, a symbolic coding tool. femLego is a toolbox for Maple which can generate complete finite element codes. Using the Maple work sheet as an interface, the relevant system equations, Navier–Stokes equations in the current case, are simply typed in together with the initial and boundary conditions, choice of extensional solvers, and optionally output formats for postprocessing. In a separate Maple worksheet, the type of finite elements, triangular P2P1 in the present case, is specified. Executing the Maple worksheet, ready-for-compile FORTRAN77 code is generated. Together with input parameters and the mesh information, the code can be immediately executed.

As mentioned by Leypoldt et al., this type of problem with azimuthal periodicity is suited to adopt the pseudospectral method in the azimuthal direction. Furthermore, with moderate strength of nonlinearity, the disturbance takes a
form of periodic waves with a distinct fundamental mode and a few harmonic ones. In fact, some experiments show that even for flow with a complex temporal oscillation, the spatial structure of the wave is still low dimensional. This allows the azimuthal structure to be resolved with sufficient accuracy even with a limited number of Fourier modes.

IV. CODE VALIDATION

The code used here was developed during this work. In order to test the correctness and rule out possible coding errors, various tests and detailed comparisons were made with results from an older code available to us (Lavalley et al.14), with satisfactory results. An example with a comparison for a subcritical case is shown in Fig. 2.

The more critical issue though is to show that the grid resolution used here is sufficient to accurately capture the flow in the particular case and parameter range we are interested in. In Table I, results from three nonuniform grids with different numbers of mesh points and resolution in the boundary layers were used. On constructing the grids, particular care was given to resolving the thin boundary layers in the hot corner region, namely, the one developing along the vertical wall and the one on the free surface. They can be characterized by the distance from the hot corner to the peak location of the surface radial velocity and the minimum surface boundary layer thickness. The region is resolved by 5 × 6, 6 × 7, and 7 × 10 P1 grid points for the 2D meshes of , and 30 × 30, for instance, in the axisymmetric computation for supercritical Ma (=39 200). As shown in Table I, the three grids result in very small differences for the base-flow computation characterized by the maximum radial velocities for both sub and supercritical Ma. Carrying out 3D simulations, the change in is still small especially between and . Therefore we adopt the grid with for the present study.

For , the 3D simulation was performed for three cases with different numbers of Fourier modes. The dependance of the results on azimuthal resolution is described in Table II with the amplitudes of the distinct Fourier modes of the temperature at the midgap on the surface. Since the results of and are almost identical, at least to capture the behavior up to third harmonic mode of the fundamental one, should be sufficient.

The mesh that is used in the simulations reported in this paper is thus the mesh on the second row of Table I.

For the grids and the range of parameters adopted in the current study, the critical Fourier mode is 3. The resulting saturated solution always exhibit a traveling wave, which is in agreement with the experiment of Shiomi et al. These features of the oscillation can be clearly seen in Fig. 3, where the energy distribution in the spatiotemporal spectral decomposition is shown using the presentation adopted by Leypoldt et al. For a mth mode oscillation, the peak frequencies can be written as , where is an integer and is the fundamental frequency of the mth mode oscillation. For an oscillation with , clockwise and counterclockwise propagating waves in the azimuthal direction are denoted by positive and negative values of n.

In the experiment in an annular configuration by Shiomi et al., it was observed that the flow shows a supercritical Hopf bifurcation where the amplitude of the oscillation increases linearly with the square root of ε in the weakly nonlinear regime. This can be confirmed in the result of the simulation shown in Fig. 4, where the evolution of the amplitudes A shows good agreement with denoted with the solid lines. The data points fall off from the lines as

<table>
<thead>
<tr>
<th>Table I. Result on computation of two-dimensional base flow for different grids.</th>
<th>Grid(N₁×N₉×N₉)</th>
<th>Δrₘᵋₙ</th>
<th>Δrₘᵋₐ</th>
<th>U₁,mₙₐₓ</th>
<th>U₂,mₙₐₓ</th>
<th>Maₑr</th>
</tr>
</thead>
<tbody>
<tr>
<td>17 × 16 × 15</td>
<td>0.0025</td>
<td>0.0042</td>
<td>0.0653</td>
<td>0.0592</td>
<td>30 548</td>
<td></td>
</tr>
<tr>
<td>21 × 16 × 17</td>
<td>0.0022</td>
<td>0.0042</td>
<td>0.0665</td>
<td>0.0593</td>
<td>28 840</td>
<td></td>
</tr>
<tr>
<td>30 × 16 × 30</td>
<td>0.0020</td>
<td>0.0033</td>
<td>0.0663</td>
<td>0.0594</td>
<td>29 078</td>
<td></td>
</tr>
</tbody>
</table>

| Table II. Magnitudes of the outstanding Fourier modes of the temperature, \( \hat{\theta}_s(\times 10^2) \) at the midgap on the surface, \((r,z) = (0.605, 1)\), for various number of Fourier modes, \(N_\theta, Ma=39 200\). \((N_\phi, N_z) = (21, 17)\). \(m\) denotes the azimuthal wavenumber. The critical wavenumber is \(m_c = 3\). | N_\theta | \(\hat{\theta}_1\) | \(\hat{\theta}_6\) | \(\hat{\theta}_9\) | \(\hat{\theta}_{12}\) |
|---|---|---|---|---|
| 8 | 0.4657 | 0.0115 | … | … |
| 16 | 1.0383 | 0.2142 | 0.0566 | 0.0131 |
| 32 | 1.0388 | 0.2147 | 0.0569 | 0.0131 |
nonlinearity of the system becomes stronger, which was also observed in the experiments for \( m = m_c \).

**V. CONTROL METHOD**

The control method is based on local modification of the surface heat flux \( q \). The basic idea is to realize a local opposition control using the knowledge of the structure of the oscillation. First, the dominant azimuthal wavenumber of the oscillation is checked. Knowing the wavenumber of the target azimuthal mode, a sensor, and an actuator are placed in different positions. Then a simple linear cancellation scheme can be realized by feeding back the sensor signal to the actuator.

The linear feedback control law can be expressed by relating an imposed local heat flux to the surface temperature at the point of the sensor as

\[
q_i(t) = q(r_0, \phi_i + d\phi_i, t) = G_1 \theta'(r_0, \phi_i, 1, t),
\]

\[ \theta' (r, \phi, z, t) = \theta(r, \phi, z, t) - \bar{\theta}(r, z, t). \]  

Here \( \phi_i \) is the azimuthal position of the \( i \)th sensor and \( d\phi_i \) is the angle between the sensor and the corresponding actuator, so that the azimuthal position of the actuator is \( \phi_i + d\phi_i \). \( r_0 = 0.605 \) is the midgap location.

The temperature fluctuation \( \theta' \) is the difference between the actual temperature and \( \bar{\theta} \), which is the time average of the spatial mean, i.e., the zeroth Fourier mode, over 5 s. \( G_1 \) is the linear control gain.

We consider two simple cancellation schemes using positive and negative \( G_1 \),

\[
d\phi_i = \begin{cases} 
\frac{2j\pi}{m_c} - \frac{\pi}{m_c}, & G_1 > 0, \\
\frac{2j\pi}{m_c}, & G_1 < 0,
\end{cases}
\]

where \( j \) is a positive integer. Negative \( G_1 \) is thus used when we expect the disturbances to be in phase at the sensor and actuator positions, and positive \( G_1 \) when they are at a 180o phase difference.

The finite size of the actuator is modeled by prescribing the spatial distribution of the heat flux from one actuator as Gaussian profiles in the radial and azimuthal direction, around the heater position \( (r_0, \phi_i + d\phi_i) \):

\[
g(r, \phi, t) = \sum_{i=1}^{p} q_i(t) \exp \left[-\left(\frac{r-r_0}{\Delta r}\right)^2 - \left(\frac{\phi - \phi_i - d\phi_i}{\Delta \phi}\right)^2\right],
\]

where \( p \) is the number of controllers. \( \Delta r \) and \( \Delta \phi \) represent the thickness and length of the actuator. \( \Delta r = 0.01 \), throughout the current study, while we have experimented with different azimuthal lengths, to be specified below.

The choice of Gaussians as heat flux profiles is not intended as a precise model of the heater in the experiments by Shiomi et al.,\(^{25}\) but rather as a means to explore the importance of the overall heater size and aspect ratio. We do, however, expect the control to be insensitive to the finer details of the heater geometry, since the amplified part of the wavenumber spectrum is restricted to rather long waves, corresponding to the length of the heater or greater. Shorter waves and higher wavenumber content in the heater model are expected to be simply filtered out by the dynamical response of the flow. We thus claim that the important features of the heater are included in Eq. (17), and that adding higher wavenumber terms to Eq. (17) would not alter the results.

**VI. RESULTS**

Numerical simulations were carried out for the annular geometry described above, with an \( A_r = H/R = 1 \) and \( H_r = R_i/R = 0.21 \). We assume microgravity conditions and disregard effects of gravity. The Prandtl number is chosen as \( Pr = 14 \) to match that of the 1 cS silicone oil used in our previous experiments. This choice of parameters makes it possible for us to compare results with those obtained experimentally by Shiomi et al.\(^{25}\) In this paper we will focus on the influence of the properties of the heaters and sensors, and we
have not attempted to cover other geometrical parameters or fluid properties. We have also neglected gravity, in order to focus on the generic aspects of control of thermocapillary convection.

The critical value of the Marangoni number for onset of oscillations $M_{ac}$ was determined by successive supercritical simulations. The oscillation amplitudes for these supercritical Marangoni numbers were extrapolated to zero, to obtain $M_{ac}$. The value obtained thus was $M_{ac}=28,840$. Notice that this value of $M_{ac}$ differs from that found by Lavallee et al., since their simulations accounted for gravity, while the present ones assume zero gravity.

We will in the remainder mainly study different attempts to control oscillation in a weakly nonlinear case with $\epsilon=0.07$ (i.e., $Ma=30,859$). Without control, the oscillation in this case shows an azimuthal wavenumber of 3. The resulting saturated solution always exhibits a traveling wave, which is in agreement with the experiment of Shiomi et al. These features of the oscillation can be clearly seen in Fig. 3, where the energy distribution in the spatiotemporal spectral decomposition at $(r,z)=(0.605,1)$ is shown using the presentation adopted by Leyholdt et al. For an $m$th mode oscillation, the peak frequencies can be written as $f_m=|n|f_{m,c}$, where $n$ is an integer and $f_{m,c}$ is the fundamental frequency of the $m$th mode oscillation. For an oscillation with $(m,|n|)$, clockwise and counterclockwise propagating waves in the azimuthal direction are denoted by positive and negative values of $n$.

A. $G_1<0$

In this case, the controllers are configured so that the temporal oscillations of the mode-3 waves are in phase at the sensor and the coupled actuator. This way, a proportional feedback control can be realized by feeding back the sensor signal to the actuator with a negative control gain.

1. Single actuator control

Experimental observations have been reported that attempts to suppress the oscillation with a single controller may result in a standing wave with nodes at the sensor location. This was investigated for the parameters $(\epsilon,\rho,\phi_1,d\phi_1,\Delta\phi,G_1)=(0.07,1,0,2\pi/3,\pi/16,-500)$. After the initial transient state, the controlled system results in a saturated oscillatory state. In Fig. 5, a sequence of pictures visualizing the isotherms on the free surface is plotted. The positioning of the sensor and heater is indicated in the small inset to the left. It can be seen that the nodes and antinodes of the wave switch their positions through axisymmetric states, which indicates that the oscillation is a mode-3 standing wave as predicted by the experimental observations. The oscillation escapes the control, since the sensor position is at a nodal line of the oscillatory mode, where the amplitude of the fluctuation is zero.

2. Multiple controller control

Now, following the idea of Petrov et al., one more controller was added to cover two degrees of freedom in the azimuthal direction. The sensors and actuators were positioned in pairs, with a $2\pi/3$ angle between the corresponding sensor and actuator, and a $\pi/2$ angle between the two sensors, is indicated in the inset in Fig. 6. In the notation adopted above the controller positions are $(\phi_1,\phi_2)=(0,\pi/2)$ and $d\phi_1=d\phi_2=2\pi/3$. This placement corresponds to that which gave the best results in the experiment of Shiomi et al.

When $\epsilon$ is very small, $\sim 0.02$, i.e., $Ma=29,500$, the oscillation can be completely suppressed to a steady axisymmetric state. In Fig. 6, the time history of the controlled oscillation at $(r,\phi,z)=(0.605,0,1)$, is shown using a gain $G_1=-1000$. The exponential decay of the oscillation shows that, in this case, the control influences the linear property of the system without influencing the stability of other modes.

On increasing $\epsilon$ slightly up to about 0.07 (i.e., $Ma=30,859$), still in the weakly nonlinear regime discussed in Shiomi et al., the controlled oscillation begins to exhibit more complexity depending on the value of $G_1$. When $G_1$ is small enough, on applying control, the oscillation still shows an exponential decay to an saturated oscillation with slightly smaller amplitude. At this point, of course, the dominant mode is still 3. On increasing $G_1$ for further suppression, although the overall magnitude of the oscillation is reduced, new modes are triggered by the control. The time history at the sensor locations $(\phi_1,\phi_2)$ are shown in Fig. 7 for a value of $G_1=-1500$. The oscillations at both locations monotonically decay until they reach the minima beyond which the oscillation grows again and eventually saturates.

The active modes contributing to the controlled oscillation can be identified by the full spectral decomposition shown in Fig. 8. The amplitude is normalized by the amplitude of the uncontrolled mode-3 traveling wave (Fig. 3) and denoted as $g$, the suppression ratio. A suppression of mode 3 and amplification of other modes can be observed. The mode with the largest energy is mode 2 which is in agreement with the implication of the experimental results. One interesting aspect of the controlled oscillation is that, even with many active modes, most of the waves are standing. The structure and the positioning of the oscillation seems to be selected for each mode to have nodes close to the controllers, at least when $\epsilon\ll 1$.

For the above case, a sequence of simulated isotherms on the free surface is shown in Fig. 9. It can be observed that triangles (a,f) and ellipses (d) appear in turns with distorted isothermal patterns in between. Maxima of the mode-3 standing wave appear when the mode-2 standing wave is in axisymmetric state and vice versa. From this, it can be understood that mode-3 and mode-2 waves have similar frequency and $\pi/4$ phase difference.

Further increase of $G_1$ results in more excitation of broad spectral components which leads to an increase of the overall magnitude of the oscillation as shown in Fig. 10. In the experiment of Barrena, the broadening of the spectra could be prevented by increasing the azimuthal length of the actuator. The idea is that, the longer the actuator is, the less energy is distributed on higher wavenumber components, therefore the excitation of the higher modes should be reduced. This aspect is explored by setting $\Delta\phi$ to $3\pi/16$, three times the original length. Correspondingly, $G_1$ is set to 2000, one third of the previous case, in order to roughly maintain...
the total output from the actuators. The spectrum shown in Fig. 11 shows a clear reduction of the broadening of both temporal and spatial spectra. Note that, in this case, the amplification of the distinct destabilized modes (1 and 2) are even larger than the case shown in Fig. 10.

B. $G_1 > 0$

In order to reduce the new wavenumber 2 mode we adopt a different configuration of the controllers, $(\phi_1, \phi_2) = (0, \pi/6)$, and $(d\phi_1, d\phi_2) = (5\pi/3, \pi/3)$ for the first and second actuator locations, as sketched in the inset in Fig. 12. This allows the sensors to be placed only an angle $\pi/6$ apart, too small to allow a wavenumber 2 mode to go undetected. This choice was also guided by the experiments in the half-zone and annular geometries by Shiomi et al.\textsuperscript{25} and Bárcena et al.\textsuperscript{29} which show that the amplification of the new appearing mode, mode 2 for this case, is weaker for the cancellation scheme with $G_1 > 0$ than with $G_1 < 0$.

A simulation for $\epsilon = 0.07$ shows that the change in the control method results in reducing the amplification of mode 2. This leads to an almost complete suppression of the oscil-
lation at the sensor locations as shown in Fig. 12. Yet, modulated signals with finite amplitudes can be detected at the sensors. The modulation has a period of about 30 s, and they are out of phase at \(f_1\) and \(f_2\). The global suppression of the oscillatory flow can be observed by the spectral decomposition shown in Figs. 13a and 13b which are time-averaged data for time of 30–40 s and 50–60 s, respectively. The two time windows correspond to when \(u_{rms}(f_1)\) and \(u_{rms}(f_2)\), where \(u_{rms}\) denotes the root mean square of \(u\). The switch in the direction of wave propagation in both of the dominant modes 2 and 3 can be observed. In both pictures, mode-2 and mode-3 waves have equivalent energy. The average suppression ratio of the dominant mode is about 15% which is in agreement with previous experiments.

The gain \(G_1\) in the control law, Eq. (14), has so far been discussed only in nondimensional terms. In order to compare with the gains found efficient in the experiments of Bárcena et al., we write down the corresponding dimensional expression for the gain:

\[
G_{1,\text{exp}} = kDTRG_1 \int_0^{2\pi} \int_{\rho_0}^1 \exp \left[ -\left( \frac{r - r_0}{\Delta r} \right)^2 \right] 
- \left( \frac{\phi - \phi_1 - d\phi}{\Delta\phi} \right)^2 \right] r dr d\phi,
\]

where \(G_{1,\text{exp}}\) is the amplification, so that \(G_{1,\text{exp}}\theta\) is the net power output at the heater. \(G_{1,\text{exp}}\) is thus measured in units of watts. \(k\) is the thermal conductivity of the fluid (W/mK), \(R\) is the radius of the container. \(\Delta r\) and \(\Delta\phi\) are the nondimensional heater widths, as used in Eq. (17). The experiments of Bárcena et al. show that, for the current range of \(\epsilon\), the necessary magnitude of the linear control gain \(G_{1,\text{exp}}\) was in the order of 0.1 W. With the values \(k=0.1\) W/m K for the 1 cS silicone oil used in the experiment, and the temperature difference \(\Delta T=20\) K from the experiment, \(G_1=3000\) from
the current simulation is equivalent to $G_{1,\text{exp}} = 0.111$ W, in good agreement with the $G_{1,\text{exp}} = 0.1$ W found in the experiment.

**Actuation with heaters**

In reality, it is not easy to construct an actuator which allows both heating and cooling. Petrov et al.\textsuperscript{21} demonstrated the possibility to use a Peltier device which heats or cools depending on the direction of the applied current. However, the magnitude of the maximum power output is limited if the time response of the actuation is to be kept within an allowable range. Therefore, in the previous experiments,\textsuperscript{29} we adopted heaters as actuators, which results in a restricted linear control law,

\begin{equation}
q(\phi_i + d\phi) = \begin{cases} 
G_1 \theta'(\phi_i), & \theta'(\phi_i) \geq 0, \\
0, & \theta'(\phi_i) < 0, 
\end{cases}
\end{equation}

when $G_1 > 0$. The influence of the restriction on the actuation was examined by simulating the same case as in Fig. 12, but limiting the actuation to heating only. Here, $G_1 = 6000$, twice the value in the previous case, in order to roughly maintain the correspondence in terms of the total power output of the actuators. The solutions of these two cases are compared in

![Image](https://example.com/fig9.png)

**FIG. 9.** A time sequence of surface isotherms from 0 to 1 of the saturated oscillation in Fig. 7. The inset to the left shows the positions of the sensors ($S_1$, $S_2$ and the heaters ($A_1$, $A_2$).
Fig. 14, where the temporal signals at $\phi = \phi_1, \phi_2$ for $G_1 = 3000$ with alternate heating and cooling (---) and $G_1 = 6000$ with only heating (----) are shown. The results show good agreement between the two cases, which encourages the use of the heaters in the experiments.

VII. CONCLUSIONS

We have studied a few representative cases of control of thermocapillary convection using numerical simulations. In addition to what was found in the previous experiments, the present simulations give us access to the complete thermal and velocity fields and their spectra. It permits us to investigate the importance of the positioning of sensors and heaters, and the influence of the properties of the heaters, in a controlled manner that complements the previous experimental findings.

On applying the cancellation scheme with negative value of $G_1$ for a very small $\varepsilon (\sim 0.02)$, the control influences the linear properties of the target mode in the system without destabilizing other modes. With a slight increase in $\varepsilon (= 0.07)$, the performance of the control is limited due to the appearance of new modes, mainly mode 2. In order to study how the spatiotemporal features of the oscillation is affected by the control we have studied azimuthal and temporal Fourier decompositions of the oscillations, easily accessible in these numerical simulations. An excess gain $G_1$ results in a broadening of the spectra. One of the facts revealed by the numerical investigation is that the control certainly induces spatial complexity to the oscillation which increases the dimension of the problem. On the other hand, it was shown that the complexity, i.e., the width of the broadened spectra depends on the azimuthal length of the actuator. This implies that adjustment of this parameter can improve the control performance.

Between the two configurations with opposite sign of $G_1$, the control works better for the one with positive $G_1$, where the two sensors can be placed only an angle $\pi/6$ apart. With this configuration, for $\varepsilon = 0.07$, the destabilization

![FIG. 10. Spectral decomposition at $(r, z) = (0.605, 1)$ for the saturated oscillation with control with high gain and short heater. $(\epsilon, p, \Delta \phi, G_1) = (0.07, 2, \pi/16, -6000)$](image1)

![FIG. 11. Spectral decomposition at $(r, z) = (0.605, 1)$ for the saturated oscillation with control, using a heater three times as long as in Fig. 10. $(\epsilon, p, \Delta \phi, G_1) = (0.07, 2, 3 \pi/16, -2000)$.](image2)

![FIG. 12. Temperature signals at the two sensor positions for the sensor positioning with positive gain. The two sensors ($S_1, S_2$) are placed at an angle of $\pi/6$, as indicated in the inset. $(r, z) = (0.605, 1)$. $(\epsilon, p, \Delta \phi, G_1) = (0.07, 2, \pi/16, 3000)$.](image3)
of new modes can be delayed and the oscillation can be suppressed significantly. A global suppression was confirmed in the entire temperature field. The obtained suppression ratio, $\gamma \sim 0.15$, matches with that obtained in the experiment.

In the previous experiments, we have blamed the low signal to noise ratio for the inferior control performance in annular configuration to that in a half zone. However, the current result suggests that the reason may lie in the difference in the phenomena themselves instead of the sensitivity of the experiment. Since the two geometries show certain differences in the flow characteristics, it is not surprising if they react differently being subjected to the same control methods.

In our previous experiments, the actuation was limited to heating only. The influence of this limitation was examined by comparing the control performance with alternate heating and cooling to that with only heating. The good agreement between the results from the two cases confirms that, even with this limitation on the actuation, the control performs in a similar manner and achieves the same extent of suppression.

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17 D. Schwabe and S. Benz, “Thermocapillary flow instabilities in an annulus


